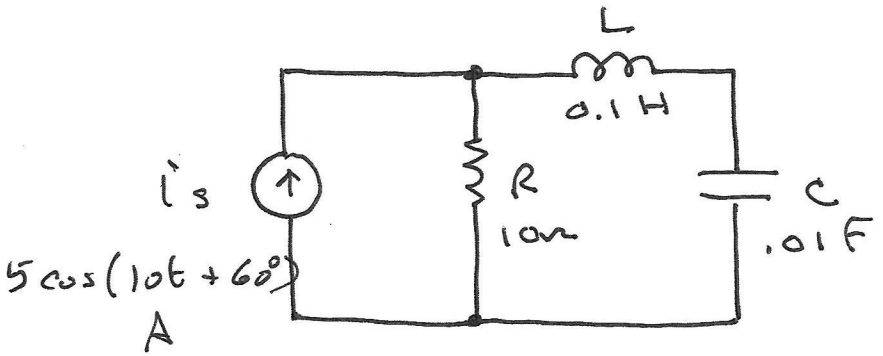
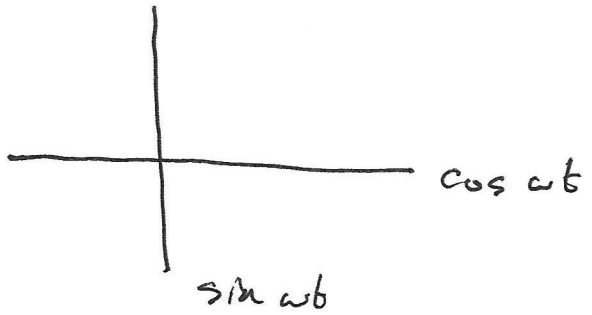
Step 1

Express the circuit in the time domain.



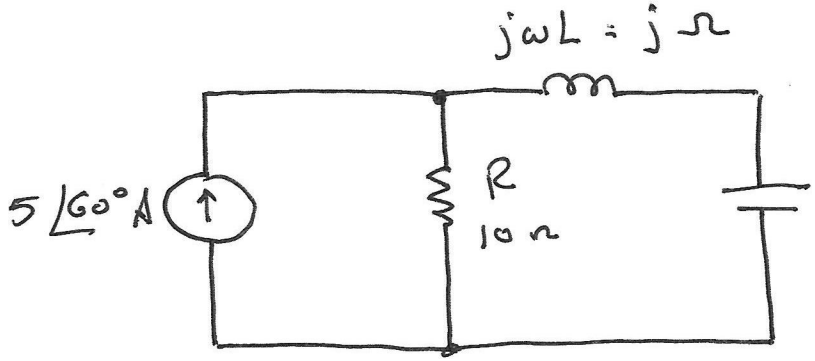


$\omega = 2\pi f$

ω: rad/s, f: Hz

Step 2

Represent the circuit in the phasor domain:



$\frac{1}{j\omega C}$ or $-\frac{j}{\omega C}$

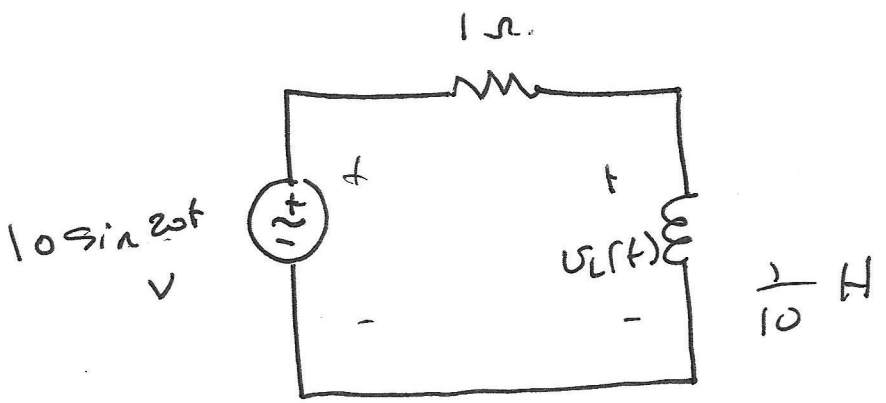
$-j10\ \Omega$

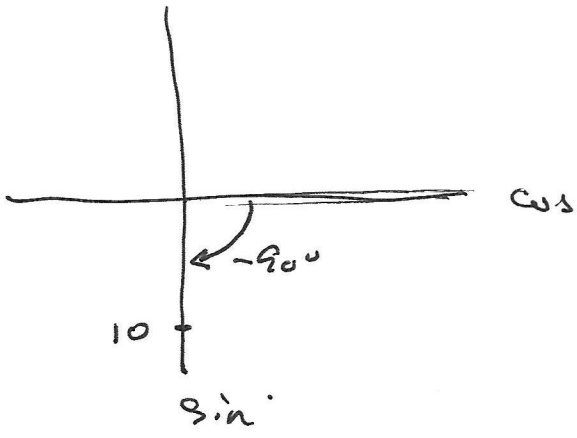
Step 3

Solve as if a resistor problem, but impedances are complex numbers. Determine the magnitude and phase angle of the variable of interest.

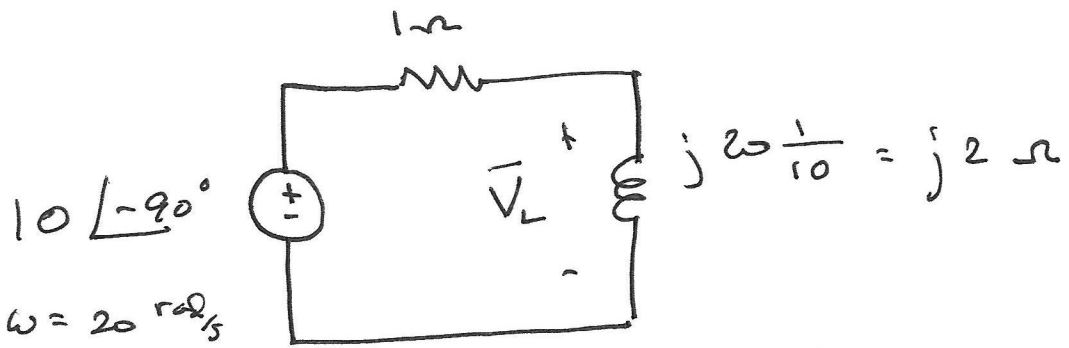
Step 4

Convert back to the time domain.





↓ convert to phasor domain



↓ solve for V_L

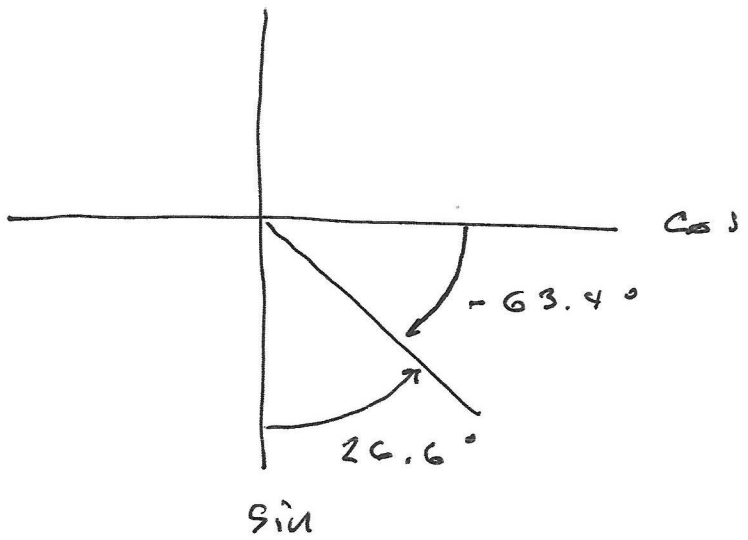
$$V_L = \frac{j2}{1+j2} \cdot 10\angle -90^\circ = \frac{2\angle 90^\circ}{\sqrt{5}\angle \tan^{-1}\frac{2}{1}} \cdot 10\angle -90^\circ$$

$$\sqrt{5}\angle 63.4^\circ$$

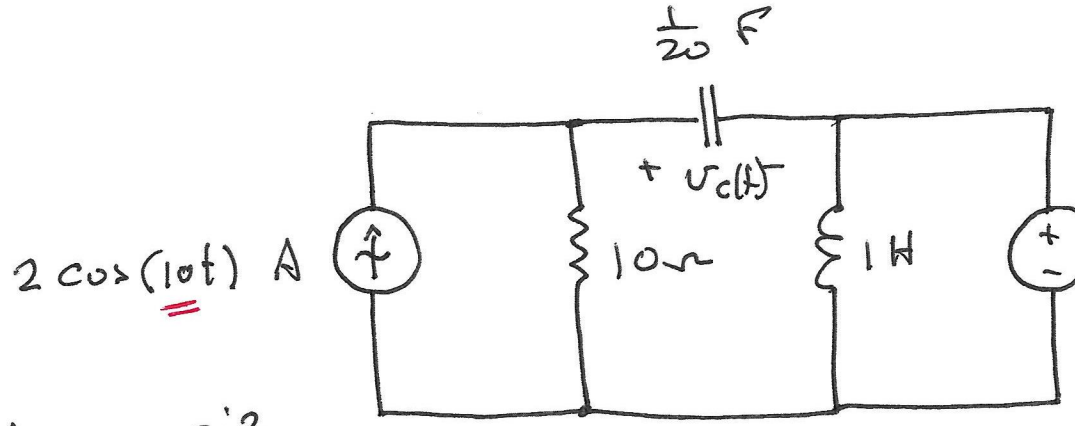
$$V_L = \frac{20}{\sqrt{5}}\angle -63.4^\circ$$

↓ convert to time domain

$$v_L(t) = \frac{20}{\sqrt{5}} \cos(20t - 63.4^\circ) \quad V$$



$$v_L(t) = \frac{20}{\sqrt{5}} \sin(20t + 26.6^\circ) \quad V$$



$2\cos(10t)\ A$

$50\sin(20t + 30°)\ V$

$= 50\cos(20t + 30° - 90°)\ V$

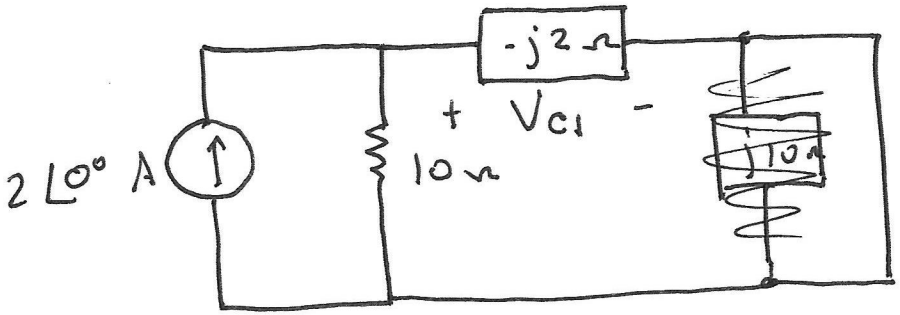
$= 50\cos(20t - 60°)\ V$

$\frac{1}{j\omega C} = \frac{1}{j10 \cdot \frac{1}{20}} = -j2$

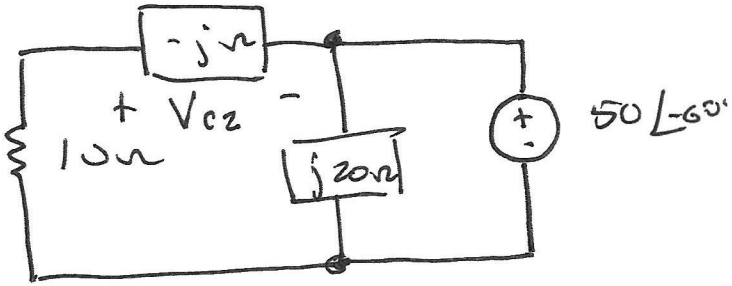
$j\omega L = j\,10 \cdot 1 = j10$

$j\omega L = j20\ \Omega$

$\frac{1}{j\omega C} = -j\ \Omega$



Solve for V_{c1}

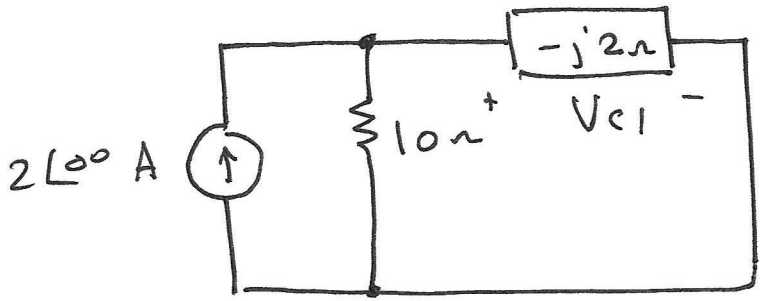


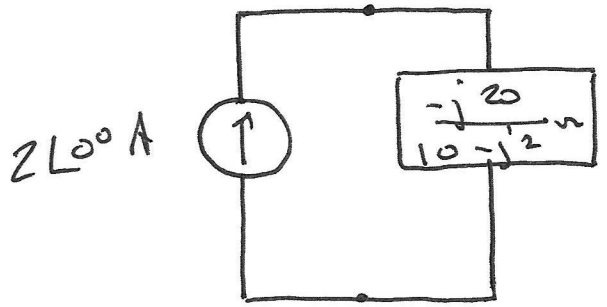
Solve for V_{c2}

Then $v_c = v_{c1} + v_{c2}$

$= |V_{c1}| \cos(10t + \angle V_{c1}) + |V_{c2}| \cos(20t + \angle V_{c2})$

To solve for V_{c1}:





$+$

V_{c1}

$-$

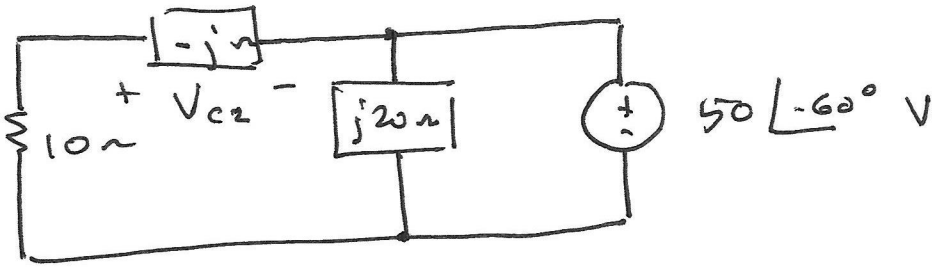
$$= \frac{-j20}{10-j2} \cdot 2\angle 0° =$$

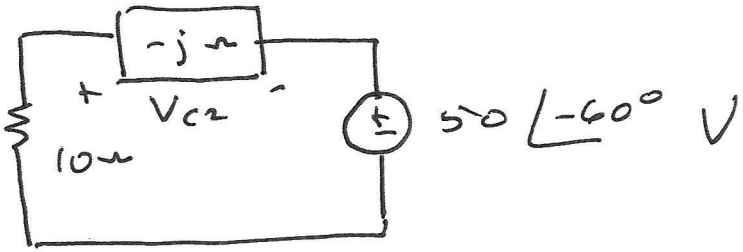
$$V_{c1} = \frac{-j20}{10-j2} \cdot \frac{10+j2}{10+j2} \; 2\angle 0°$$

$$= \frac{80-j400}{104} = \frac{10}{13} - j\frac{50}{13} \text{ V}$$

$$v_{c1}(t) = \sqrt{\left(\frac{10}{13}\right)^2 + \left(\frac{50}{13}\right)^2} \angle \tan^{-1}(-5) \text{ V}$$

To solve for V_{c2}.

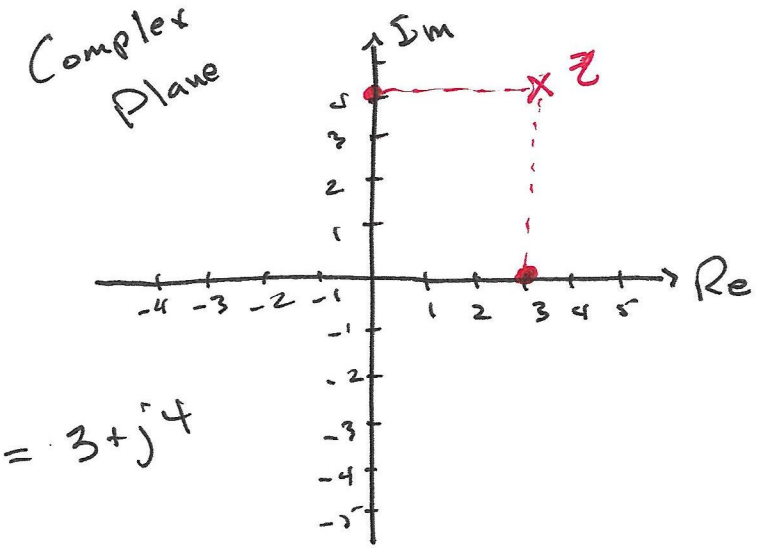




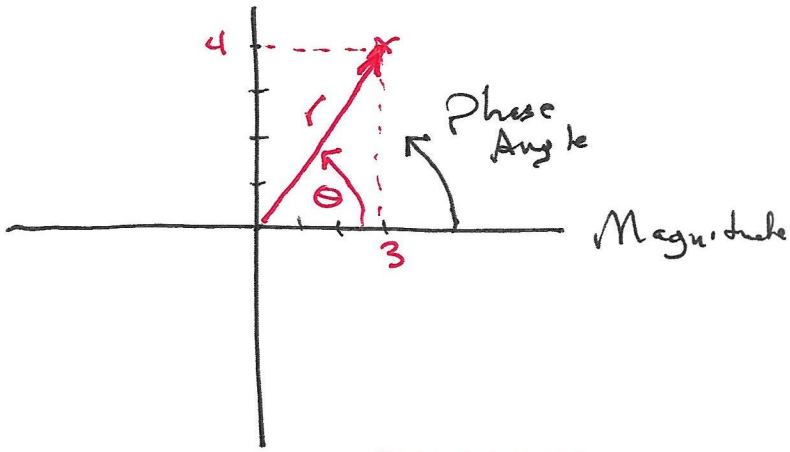
$$V_{c2} = -\frac{-j1}{10-j} \cdot 50\angle -60^\circ$$

$$= \frac{j \cdot 50\angle -60^\circ}{10-j} \cdot \frac{10+j}{10+j}$$

$$= \frac{50\angle -60^\circ \ (-1+j10)}{101}$$



$z = 3 + j4$



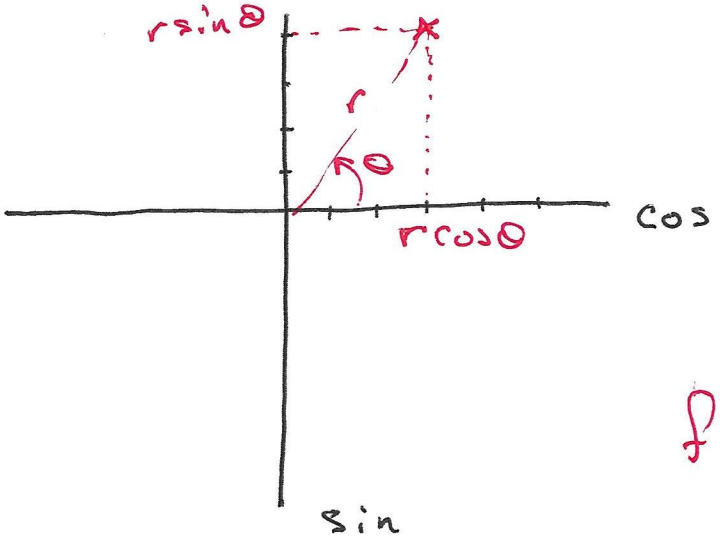
$$r = \sqrt{3^2 + 4^2}$$

$$= 5$$

$$\theta = \tan^{-1}\frac{4}{3}$$

$$= 53.13^\circ$$

$$z = 5 \angle 53.13^\circ$$



$$f(t) = 5\cos(10t + 53.13^\circ)$$

$$= 5\cos(53.13^\circ)\cos 10t - 5\sin(53.13^\circ)\sin 10t$$